MATH 1105 - FALL 2008 - 11-24-08
SECTION 2, QUIZ 3

You have 30 minutes to complete the following problems. No notes or calculators are allowed or necessary. This quiz is out of 35 points. Good Luck!

(1) (8 Points) Of drivers over the age of 65 that are getting their licence renewed, 80 % pass the eye exam.

a. If 100 drivers over the age of 65 take the eye exam, what is the probability that exactly 76 of them pass? Do **not** simplify your answer.

$\binom{100}{76}.8^{76}.2^{24}$

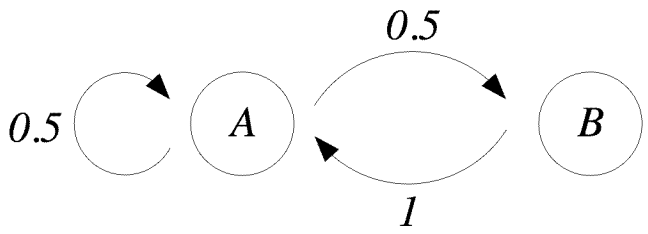
b. Use the normal approximation and the table below to answer the following: What is the probability that greater than 76 of the drivers pass the eye exam?

Area under the standard normal curve:

z	Area to left of z
-3	0.0013
-2	0.0228
-1	0.1587
0	0.5
1	0.8413
2	0.9772
3	0.9987

$\mu = (100)(.8) = 80$, $\sigma = \sqrt{(100)(.8)(.2)} = 4$
$z = \frac{76-80}{4} = -1$
$P(x > 76)$ = (Area to the right of $x = 76$) = (Area to the right of $z = -1$ in standard normal curve) $= 1 -$ (Area to the right of $z = -1$ in standard normal cruve) $= 1 - 0.1587 = 0.8413$.

(2) (12 Points) Consider the transition diagram for a Markov chain with two states, A and B, shown below.



a. What is this transition matrix for this Markov chain?

$$\begin{bmatrix} .5 & .5 \\ 1 & 0 \end{bmatrix}$$

b. Is this Markov chain regular? Justify your answer.

$$\begin{bmatrix} .5 & .5 \\ 1 & 0 \end{bmatrix}^2 = \begin{bmatrix} .75 & .25 \\ .5 & .5 \end{bmatrix}$$

The Markov chain is regualr as the transition matrix squared has all positive entries.

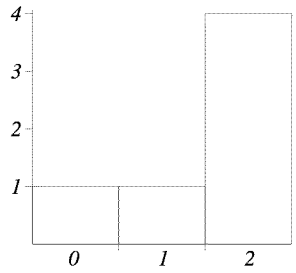
c. Does the Markov chain have an equilibrium vector? If so, calculate it. If not, explain why not.

The equilibrium vector os the solution to the following system of equations:
$x + y = 1$
$.5x + y = x$
$.5x = y$
Substituting the last equation into the first shows that $x = \frac{2}{3}$, and the solving for y yields $y = \frac{1}{3}$. The equilibrium vector is

$$\begin{bmatrix} \frac{2}{3} & \frac{1}{3} \end{bmatrix}.$$

(3) (15 Points) There are 4 balls in a bag, 2 red and 2 green.

a. Draw a histogram of the probability distribution for the number of red balls taken when 2 balls are taken from the bag without replacement.

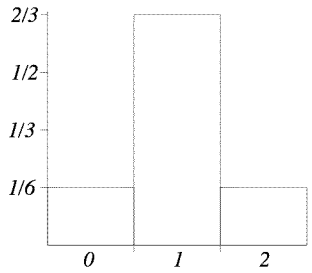


Note $P(x=0) = \frac{\binom{2}{2}}{\binom{4}{2}} = \frac{1}{6}$, $P(x=1) = \frac{\binom{2}{1}\binom{2}{1}}{\binom{4}{2}} = \frac{2}{3}$, and $P(x=2) = \frac{\binom{2}{2}}{\binom{4}{2}} = \frac{1}{6}$.

b. What is the expected value of this probability distribution?

$E(x) = 0(\frac{1}{6}) + 1(\frac{2}{3}) + 2(\frac{1}{6}) = 1.$

c. Suppose that 6 people each took 2 balls out of the bag, and the balls were replaced after each person. The number of red balls taken by the people were 0, 1, 2, 2, 2, and 2. Draw a histogram for this frequency distribution.



d. Calculate the mean, median, mode, and standard deviation of this frequency distribution. Do **not** simplify the standard deviation.

Mean $= \frac{0(1)+1(1)+2(4)}{6} = \frac{3}{2}$, Median $= 2$, Mode $= 2$, Standard Deviation $= \sqrt{\frac{0^2+1^2+4(2^2)-6(\frac{3}{2}^2)}{5}}$.

Bonus. (3 Points) Consider a regular Markov chain with transition matrix P. Suppose that the Markov chain with transition matrix P^{37812} has equilibrium vector

$$\left[\begin{array}{ccc} \frac{3}{7} & \frac{2}{7} & \frac{2}{7} \end{array}\right].$$

What is the equilibrium vector of the Markov chain with transition matrix P? Justify your answer, the correct answer without correct justification receives no credit.

Note that since P is regular, it must have an equilibrium vector, and if the rows of P^n converged to something other than $\left[\begin{array}{ccc} \frac{3}{7} & \frac{2}{7} & \frac{2}{7} \end{array}\right]$ then the rows of $(P^{37812})^n$ could not converge to $\left[\begin{array}{ccc} \frac{3}{7} & \frac{2}{7} & \frac{2}{7} \end{array}\right]$. So $\left[\begin{array}{ccc} \frac{3}{7} & \frac{2}{7} & \frac{2}{7} \end{array}\right]$ must be the equilibrium vector of P. In general, if P is regular taking powers of P cannot alter the equilibrium vector.